$$ds^2 = e^{2A(w)} \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{Minkowski metric}} + dw^2$$

Christoffel symbols

$$g_{ab} = \begin{pmatrix} -e^{2A} & & & & \\ & e^{2A} & & & \\ & & e^{2A} & & \\ & & & e^{2A} & \\ & & & & 1 \end{pmatrix} \qquad g^{ab} = \begin{pmatrix} -e^{-2A} & & & & \\ & e^{-2A} & & & \\ & & e^{-2A} & & \\ & & & e^{-2A} & \\ & & & & 1 \end{pmatrix}$$

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{cb} \right)$$

$$a, b, c, \ldots (= \mu, w) = 0, 1, 2, 3, w$$

$$\Gamma^0_{bc} = -\frac{1}{2} e^{-2A} \left(\partial_b g_{0c} + \partial_c g_{0b} - \cancelto{0}{\partial_0 g_{cb}} \right)$$

$$= \frac{1}{2} e^{-2A} \left(\delta_{bw}\delta_{c0} + \delta_{b0}\delta_{wc} \right) \left(e^{2A} \right)'$$

$$\boxed{\Gamma^0_{0w} = A' = \frac{dA}{dw}} \qquad \text{any other } \Gamma^0_{\cdot\cdot} = 0$$

$$\Gamma^i_{bc} = \frac{1}{2} e^{-2A} \left(\partial_b g_{ic} + \partial_c g_{ib} - \cancelto{0}{\partial_i g_{cb}} \right)$$

$$= \frac{1}{2} e^{-2A} \left(\delta_{ci}\delta_{bw} + \delta_{bi}\delta_{cw} \right) \left(e^{2A} \right)'$$

$$\boxed{\Gamma^i_{wj} = A' \delta^i_j}$$

$$\Gamma^w_{bc} = \frac{1}{2} \left(\cancelto{0}{\partial_b g_{wc}} + \cancelto{0}{\partial_c g_{bw}} - \partial_w g_{bc} \right) \Rightarrow \boxed{\Gamma^w_{00} = e^{2A} A'} \quad \boxed{\Gamma^w_{ij} = -e^{2A} A' \delta_{ij}}$$

Ricci tensor

Transformation properties of R_{ab} under $x^\mu \to -x^\mu$ imply that the only nonzero components are R_{00}, R_{ii}, R_{ww}.

Recall that

$$R_{ab} = \partial_c \Gamma^c_{ab} - \partial_b \Gamma^c_{ac} + \Gamma^d_{dc}\Gamma^c_{ba} - \Gamma^c_{bd}\Gamma^d_{ca}$$

$$\Gamma^a_{ab} = \frac{1}{\sqrt{-g}}\partial_b \sqrt{-g} = e^{-4A}\partial_b e^{4A} = 4A'\delta_{bw}$$

It follows

$$R_{00} = \partial_w \Gamma^w_{00} - 0 + \underbrace{\Gamma^b_{bw}\Gamma^w_{00}}_{4A' e^{2A} A'} - \underbrace{\Gamma^c_{0b}\Gamma^b_{c0}}_{= \Gamma^w_{00}\Gamma^0_{w0} + \Gamma^0_{0w}\Gamma^w_{00} = 2e^{2A}A'^2}$$

$$R_{00} = (e^{2A}A')' + 4e^{2A}A'^2 - 2e^{2A}A'^2$$

$$\boxed{R_{00} = e^{2A}(A'' + 4A'^2)}$$

$$R_{ii} = \partial_w \Gamma^w_{ii} - 0 + \underbrace{\Gamma^a_{aw}\Gamma^b_{ii}}_{4A'(-e^{2A}A')} - \underbrace{\Gamma^a_{ib}\Gamma^b_{ai}}_{= \Gamma^i_{iw}\Gamma^w_{ii} + \Gamma^w_{ii}\Gamma^i_{wi} = 2(-e^{2A}A')} \leftarrow \left\{ \begin{array}{l}\text{No sum}\\ \text{over } i\end{array}\right.$$

$$R_{ii} = (-e^{2A}A')' - 4e^{2A}A'^2 + 2e^{2A}A'^2$$

$$\boxed{R_{ii} = -e^{2A}(A'' + 4A'^2)}$$

$$R_{ww} = 0 - \partial_w \Gamma^\alpha_{w\alpha} + 0 - \underbrace{\Gamma^\alpha_{w\beta}\Gamma^\beta_{\alpha w}}_{\Gamma^0_{w0}\Gamma^0_{0w} + \Gamma^i_{wj}\Gamma^j_{iw}}$$

$$\Gamma^0_{w0}\Gamma^0_{0w} + \Gamma^i_{wj}\Gamma^j_{iw} = A'^2 + A'^2 \underbrace{\delta^i_j \delta^j_i}_{3 \text{ for our metric}} = 4A'^2$$

$$R_{ww} = -(4A')' - 4A'^2$$

$$\boxed{R_{w} = -4A'' - 4A'^2}$$

$$R = g^{ab}R_{ab} = -e^{2A}R_{tt} + \sum_{i=1}^{3} e^{2A} R_{ii} + R_{ww}$$

$$= -4(A'' + 4A'^2) + (-4A'' - 4A'^2)$$

$$\boxed{R = -8A'' - 20A'^2}$$

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$$

$$G_{00} = e^{2A}\left[A'' + 4A'^2 - \frac{1}{2}(8A'' + 20A'^2)\right]$$

$$G_{00} = e^{2A}(-3A'' - 6A'^2)$$

$$G_{ii} = e^{2A}\left[-A'' - 4A'^2 + \frac{1}{2}(8A'' + 20A'^2)\right]$$

$$G_{ii} = e^{2A}(3A'' + 6A'^2)$$

$$G_{ww} = -4A'' - 4A'^2 + \frac{1}{2}(8A'' + 20A'^2)$$

$$G_{ww} = 6A'^2$$

<u>Einstein equations</u> : $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$

(tt) $\quad -3A'' - 6A'^2 - \Lambda = 0$

(ii) $\quad 3A'' + 6A'^2 + \Lambda = 0$

ww $\quad 6A'^2 + \Lambda = 0 \quad \Rightarrow \quad \Lambda = -6A'^2 < 0 \quad \Rightarrow$

$\boxed{\Lambda \text{ negative}}$

$\Rightarrow \boxed{A'' = 0}$

If $\Lambda = 0 \Rightarrow A = \text{cnst} \Rightarrow$ Minkowski metric in 5 dim

$\Uparrow$

Rescale $\quad x^\mu e^A \to x^\mu$

Problema propuesto para entregar en enero de 2015 como evaluación continua (y entregado por casi todos).